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STABILITY OF AN ELASTIC SYSTEM SUBJECTED TO A
CIRCULATORY FORCE
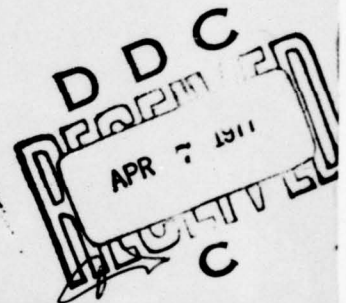
G. L. Anderson

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The stability of a vertical double pendulum with elastic joints, mounted on a platform whose horizontal motion is constrained by an elastic spring and subjected to circulatory force, is examined. Attention is focused on the determination of the critical loads of divergence and flutter as functions of the platform mass and spring stiffness parameters, which characterize the effects of the support on the stability of the double pendulum. The frequency equation is derived, and several typical eigencurves are plotted in the load-frequency plane. Stability maps in the load-tangency coefficient plane indi-

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Block 20 (continued)

cate the existence of regions of stability, divergence and flutter. Certain boundary curves of these regions are strongly dependent upon the values of the parameters that characterize the support.

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1. INTRODUCTION

For a double pendulum subjected to a circulatory force applied at its free end, Herrmann and Bungay [1] determined the dependence of the critical loads of divergence and flutter upon a tangency coefficient, α, which specified the orientation of the applied force relative to the instantaneous deformed configuration of the system. It was assumed in reference [1] that elastic hinges at the joints in the double pendulum exerted linear restoring moments $c\phi_1$ and $c(\phi_2-\phi_1)$, where c denotes the stiffness of the hinges and ϕ_1 and ϕ_2 are angles that specify the configuration of the system. To study the influence of support flexibility on the state of stability of this system, Sugiyama, et al [2], assumed that the linear restoring moments exerted by the hinges were represented as $c_1\phi_1$ and $c_2(\phi_2-\phi_1)$. They then derived expressions for the critical loads as functions of the ratio of stiffnesses, c_1/c_2.

The objective of the present investigation is to consider again the effect of the characteristics of the support on the stability of the system, but here it will be assumed that the support consists of a platform of mass M that is attached to a horizontal elastic spring of stiffness and that is constrained to move on a smooth horizontal surface. As a result of the introduction of this additional mass and degree of freedom, the system to be studied is, of necessity, one of

1. Herrmann, G., and Bungay, R. W., "On the Stability of Elastic Systems Subjected to Nonconservative Forces," Journal of Applied Mechanics, Vol 31, 1964, pp. 435-440.

2. Sugiyama, Y., Maeda, S., and Kawagoe, H., "Destabilizing Effect of Elastic Constraint on the Stability of Nonconservative Elastic Systems," Theoretical and Applied Mechanics, Vol 22, University of Tokyo Press, Tokyo, 1974, pp. 33-45.

three degrees of freedom. The frequency equation for the system is derived, and equations for the critical loads of flutter and divergence are derived. Stability maps in the load-tangency plane reveal regions of stability, divergence, and flutter, some of whose boundary curves are strongly dependent upon the values of the mass and stiffness parameters M and k.

2. THE EQUATIONS OF MOTION

Consider the system shown in Figure 1 that comprises a double pendulum mounted on a platform of mass M that is constrained to move on a smooth horizontal surface. The configuration of the system is specified by the coordinate $x_1(t)$, which locates the center of mass of the platform relative to a fixed origin O' and by the angles $\phi_1(t)$, $\phi_2(t)$ formed between the vertical and each of the two bars in the double pendulum. The double pendulum consists of two rigid, weightless bars of equal length ℓ and carries concentrated masses m_1 and m_2 located at distances a_1 and a_2 from the hinges O and A, respectively. The hinged joints at O and A exert linear elastic restoring moments $c\phi_1$ and $c(\phi_2-\phi_1)$, where c denotes a torsional spring constant, and the horizontal motion of the platform is restrained by an extensible spring of stiffness k, which is fastened to a rigid wall at C. A force of magnitude P, which acts at an angle $\alpha\phi_2$, relative to the vertical, where α designates the tangency coefficient, is applied at the free end B of the pendulum. This applied force is a circulatory force when $\alpha \neq 0$ and a conservative force only when $\alpha = 0$. Moreover, it is said to be tangential if $\alpha = 1$, sub-tangential if $0<\alpha<1$, super-tangential if $\alpha>1$, and anti-tangential if $\alpha<0$.
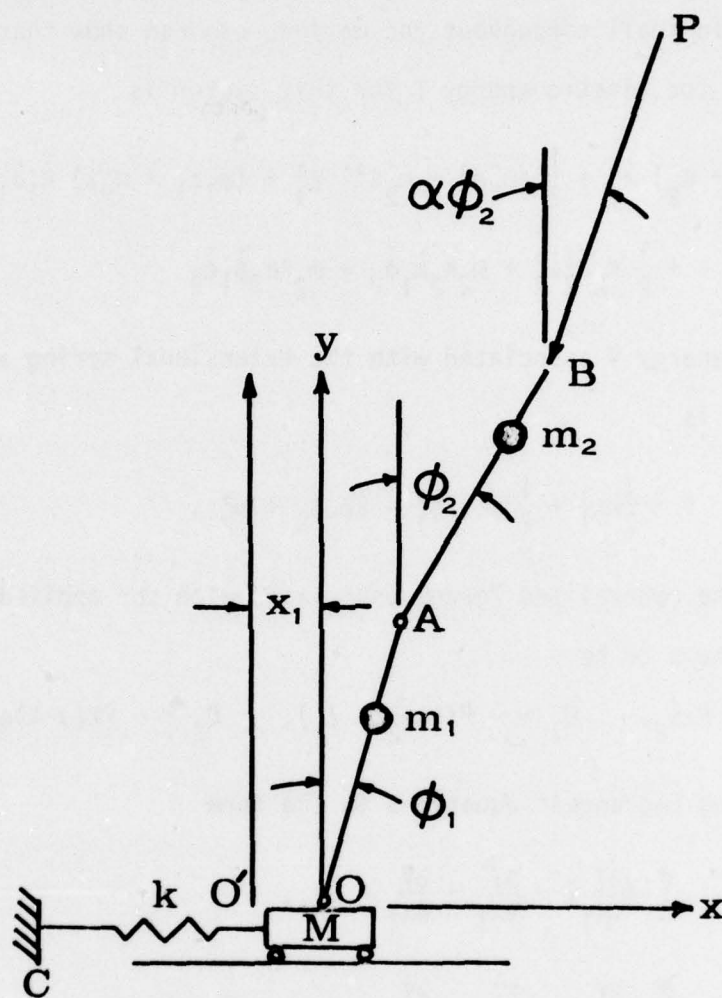
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Figure 1. A double pendulum mounted on a movable base and subjected to a circulatory force.

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Under the assumption that the displacement x_1 and the angles ϕ_1 and ϕ_2 remain small throughout the motion, one can show that the expression for the kinetic energy T for this system is

$$T = \frac{1}{2}(M + m_1 + m_2)\,\dot{x}_1^2 + \frac{1}{2}(m_1 a_1^2 + m_2 \ell^2)\,\dot{\phi}_1^2 + (m_1 a_1 + m_2 \ell)\,\dot{x}_1 \dot{\phi}_1 +$$

$$+ \frac{1}{2} m_2 a_2^2 \dot{\phi}_2^2 + m_2 a_2 \dot{x}_1 \dot{\phi}_2 + m_2 \ell a_2 \dot{\phi}_1 \dot{\phi}_2 \qquad (1)$$

The potential energy V associated with the extensional spring and the elastic joints is

$$V = \frac{1}{2}k x_1^2 + \frac{1}{2}c\,(2\phi_1^2 - 2\phi_1\phi_2 + \phi_2^2)\,. \qquad (2)$$

Furthermore, the generalized forces associated with the applied force P are easily shown to be

$$Q x_1 = -P\alpha\phi_2, \qquad Q_1 = -P\ell(\alpha\phi_2 - \phi_1), \qquad Q_2 = -P\ell(\alpha-1)\phi_2. \qquad (3)$$

Therefore, using Lagranges' equations in the form

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{x}_1}\right) - \frac{\partial T}{\partial x_1} + \frac{\partial V}{\partial x_1} = Q x_1,$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\phi}_n}\right) - \frac{\partial T}{\partial \phi_n} + \frac{\partial V}{\partial \phi_n} = Q_n, \quad n = 1,2,$$

with equations (1) - (3), one obtains the equations of motion

$$(M + m_1 + m_2)\ddot{x}_1 + k x_1 + (m_1 a_1 + m_2 \ell)\ddot{\phi}_1 + m_2 a_2 \ddot{\phi}_2 + P\alpha\phi_2 = 0, \qquad (4)$$

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$$(m_1 a_1 + m_2 \ell)\ddot{x}_1 + (m_1 a_1^2 + m_2 \ell^2)\ddot{\phi}_1 + (2c - P\ell)\phi_1 +$$

$$+ m_2 \ell a_2 \ddot{\phi}_2 - (c - \alpha P\ell)\phi_2 = 0, \tag{5}$$

$$m_2 a_2 \ddot{x}_1 + m_2 \ell a_2 \ddot{\phi}_1 - c\phi_1 + m_2 a_2^2 \ddot{\phi}_2 + [c - P\ell(1 - \alpha)]\phi_2 = 0 . \tag{6}$$

It is desirable to express equations (4) - (6) in dimensionless form. This may be accomplished by making the changes of dependent and independent variables $x_4(t) = \ell x(\tau)$ and $t = \tau\ell(m/c)^{1/2}$, respectively, and by defining

$$\kappa = \frac{k\ell^2}{c}, \quad Q = \frac{P\ell}{c}, \quad M = m\mu, \quad m_n = m\mu_n, \quad a_n = \alpha_n\ell, \quad n = 1,2.$$

Hence, equations (4) - (6) can be written as

$$(\mu_1 + \mu_2 + \mu)\ddot{x} + \kappa x + (\mu_1\alpha_1 + \mu_2)\ddot{\phi}_1 + \mu_2\alpha_2\ddot{\phi}_2 + \alpha Q\phi_2 = 0, \tag{7}$$

$$(\mu_1\alpha_1 + \mu_2)x + (\mu_1\alpha_1^2 + \mu_2)\ddot{\phi}_1 + (2 - Q)\phi_1 + \mu_2\alpha_2\ddot{\phi}_2 - (1-\alpha Q)\phi_2 = 0 \tag{8}$$

$$\mu_2\alpha_2\ddot{x} + \mu_2\alpha_2\ddot{\phi}_1 - \phi_1 + \mu_2\alpha_2^2\ddot{\phi}_2 + [1 - (1 - \alpha)Q]\phi_2 = 0, \tag{9}$$

where now $\dot{x} = dx/d\tau$, etc.

3. THE PURELY DYNAMICAL SYSTEM

Suppose next that the masses m_1 and m_2 are located at the ends of the two bars in the double pendulum, i.e., at the points A and B, respectively, in Figure 1. This immediately implies that $\alpha_1 = \alpha_2 = 1$, so that equations (7) - (9) become

$$(\mu_1 + \mu_2 + \mu)\ddot{x} + \kappa x + (\mu_1 + \mu_2)\ddot{\phi}_1 + \mu_2\ddot{\phi}_2 + \alpha Q\phi_2 = 0, \tag{10}$$

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$$(\mu_1 + \mu_2)\ddot{x} + (\mu_1 + \mu_2)\ddot{\phi}_1 + .(2 - Q)\phi_1 + \mu_2\ddot{\phi}_2 - (1 - \alpha Q)\phi_2 = 0, \quad (11)$$

$$\mu_2\ddot{x} + \mu_2\ddot{\phi}_1 - \phi_1 + \mu_2\ddot{\phi}_2 + [1 - (1 - \alpha)Q]\phi_2 = 0. \quad (12)$$

These differential equations with constant coefficients admit a solution of the form

$$x(\tau) = X_1 e^{i\omega\tau}, \quad \phi_n(\tau) = X_{n+1} e^{i\omega\tau}, \quad n = 1,2, \quad i = (-1)^{1/2}, \quad (13)$$

where the X_j's are constants and ω denotes the dimensionless frequency parameter for the system. Substitution of equation (13) into equations (10) - (12) yields the set of homogeneous algebraic equations

$$\sum_{n=1}^{3} [Amn\omega^2 - Cmn - QDmn]Xn = 0, \quad m = 1,2,3 \quad (14)$$

where

$$
\begin{array}{lll}
A_{11} = \mu_1 + \mu_2 + \mu, & A_{12} = \mu_1 + \mu_2, & A_{13} = \mu_2, \\
A_{21} = \mu_1 + \mu_2, & A_{22} = \mu_1 + \mu_2, & A_{23} = \mu_2, \\
A_{31} = \mu_2, & A_{32} = \mu_2, & A_{33} = \mu_2, \\
C_{11} = \kappa, & C_{12} = 0, & C_{31} = 0, \\
C_{21} = 0, & C_{22} = 2, & C_{32} = -1, \\
C_{31} = 0, & C_{32} = -1, & C_{33} = 1, \\
D_{11} = 0, & D_{12} = 0, & D_{13} = \alpha, \\
D_{21} = 0, & D_{22} = -1, & D_{23} = \alpha, \\
D_{31} = 0, & D_{32} = 0, & D_{33} = -(1-\alpha).
\end{array}
\quad (15)
$$

It is now easily shown that the frequency equation associated with equation (14) is

$$p_0\omega^6 - p_2\omega^{4\cdot} + p_4\omega^2 - p_6 = 0, \tag{16}$$

where

$$p_0 = \mu_1\mu_2\mu,$$

$$p_2 = (2 + \kappa)\mu_1\mu_2 + \mu(\mu_1 + 5\mu_2) - Q[\mu_2(\mu_1 + 2\mu) + \mu\mu_1 (1 - \alpha)],$$

$$p_4 = \mu_1(1 + \kappa) + \mu_2(1 + 5\kappa) + \mu - Q[(\mu_1\kappa + 3\mu)(1 - \alpha) + \mu_1(3 - 2\alpha) +$$

$$+ \mu_2 (3 + 2\kappa)] + Q^2[(\mu + \mu_1)(1 - \alpha) + \mu_2],$$

$$p_6 = \kappa[1 - 3(1 - \alpha) Q + (1 - \alpha)Q^2]. \tag{17}$$

It may be noted that the value of the determinant of the dimensionless inertia matrix, $\underset{\sim}{A} = (A_{mn})$, is $\det (\underset{\sim}{A}) = p_0 = \mu_1\mu_2\mu$, i.e., the product of the three fundamental mass parameters associated with the system. If none of these mass parameters is zero, then the inertia matrix is non-singular, and the system is said to be a <u>purely dynamic system</u> [3].

For the sake of being specific, suppose now that $\mu_2 = 2$ and $\mu_2 = 1$, so that equation (17) becomes

$$p_0 = 2\mu, \quad p_2 = 4 + 2\kappa + 7\mu - 2Q[1 + \mu(2 - \alpha)],$$

$$p_4 = 3 + 7\kappa + \mu - Q[(4 + 2\kappa + 3\mu)(1 - \alpha) + 5 + 2\kappa] +$$

$$+ Q^2[(2 + \mu)(1 - \alpha) + 1],$$

$$p_6 = \kappa[1 - 3(1 - \alpha)Q + (1 - \alpha)Q^2]. \tag{18}$$

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3. Ku, A. B., "On the Stability of a Linear Nongyroscopic Conservative System," <u>Zeitschrift für angewandte Mathematic und Physik</u>, Vol 20, 1977, pp. 986-991.

If the system becomes unstable by divergence, the critical loads Q_{bj}, $j = 1,2$, can be computed from the condition of vanishing frequency. Putting $\omega = 0$ in equation (16), one obtains $p_6 = 0$, whence, by virtue of the last relationship in equation (18),

$$(1 - \alpha)Q_b^2 - 3(1 - \alpha)Q_b + 1 = 0, \tag{19}$$

provided that $\kappa \neq 0$. But equation (19) is identical to the expression for the divergence loads of a double pendulum attached to an immovable base, which has been discussed by Herrmann and Bungay [1]. Clearly, the mass M of the platform on which the double pendulum is mounted and the constraining extensional spring of stiffness k have no influence whatsoever on the values of Q_{bj} for the system of three degrees of freedom under consideration here.

The flutter loads Q_e for the system can be determined from the condition of the merging of two natural frequencies of the system. Because the frequency equation (16) may be considered a cubic polynomial in ω^2, the condition for the coalescence of two of its roots is, according to reference [4],

$$p_0^2[4p_0p_4^3 - p_2^2p_4^2 + 27p_0^2p_6^2 - 18p_0p_2p_4p_6 + 4p_2^3p_6] = 0. \tag{20}$$

If $p_0 \neq 0$, this condition becomes

$$4p_0p_4^3 - p_2^2p_4^2 + 27p_0^2p_6^2 - 18p_0p_2p_4p_6 + 4p_2^3p_6 = 0. \tag{21}$$

1. Herrmann, G., and Bungay, R. W., "On the Stability of Elastic Systems Subjected to Nonconservative Forces," Journal of Applied Mechanics, Vol 31, 1964, pp. 435-440.

4. Walter, W. W. and Anderson, G. L., "Stability of a System of Three Degrees of Freedom Subjected to a Circulatory Force," submitted for publication.

Substitution of equation (18) into equation (21) yields the following sextic for the critical flutter load Q_e:

$$\sum_{n=0}^{6} r_n Q_e^n = 0, \tag{22}$$

where the r_n's, which will not be reproduced here, are lengthy polynomials in the mass parameter μ, the stiffness parameter κ, and the tangency coefficient α.

4. NUMERICAL RESULTS FOR THE PURELY DYNAMICAL SYSTEM

Equations (19) and (22) may be solved numerically for the critical loads of divergence and flutter, respectively, for given values of the parameters α, μ, and κ. The objective of these numerical calculations consisted of the preparation of stability maps, i.e., plots in the $Q\alpha$-plane for selected values of μ and κ. The curves in the $Q\alpha$=plane obtained from equations (19) and (22) form the boundaries of the various regions of stability, divergence, flutter, and divergence-flutter. In order to identify the system's behavior in a given region, it was necessary to examine several eigencurves, i.e., plots in the $Q\omega^2$-plane. Three typical examples of are shown in Figure 2 for (a) $\alpha = 0.3$, (b) $\alpha = 1.125$, and (c) $\alpha = 1.175$ with $\mu = 1$, $\kappa = 1/10$. In these eigencurves, one can observe the points of coalescence of various modes, which indicate either the onset or cessation of a state of flutter, and the intersection of the curves with the load axis, which indicates, in the cases depicted, the transition of a frequency from an imaginary value to a real value. Whenever a frequency for a particular mode is such that $\omega^2 < 0$, it is
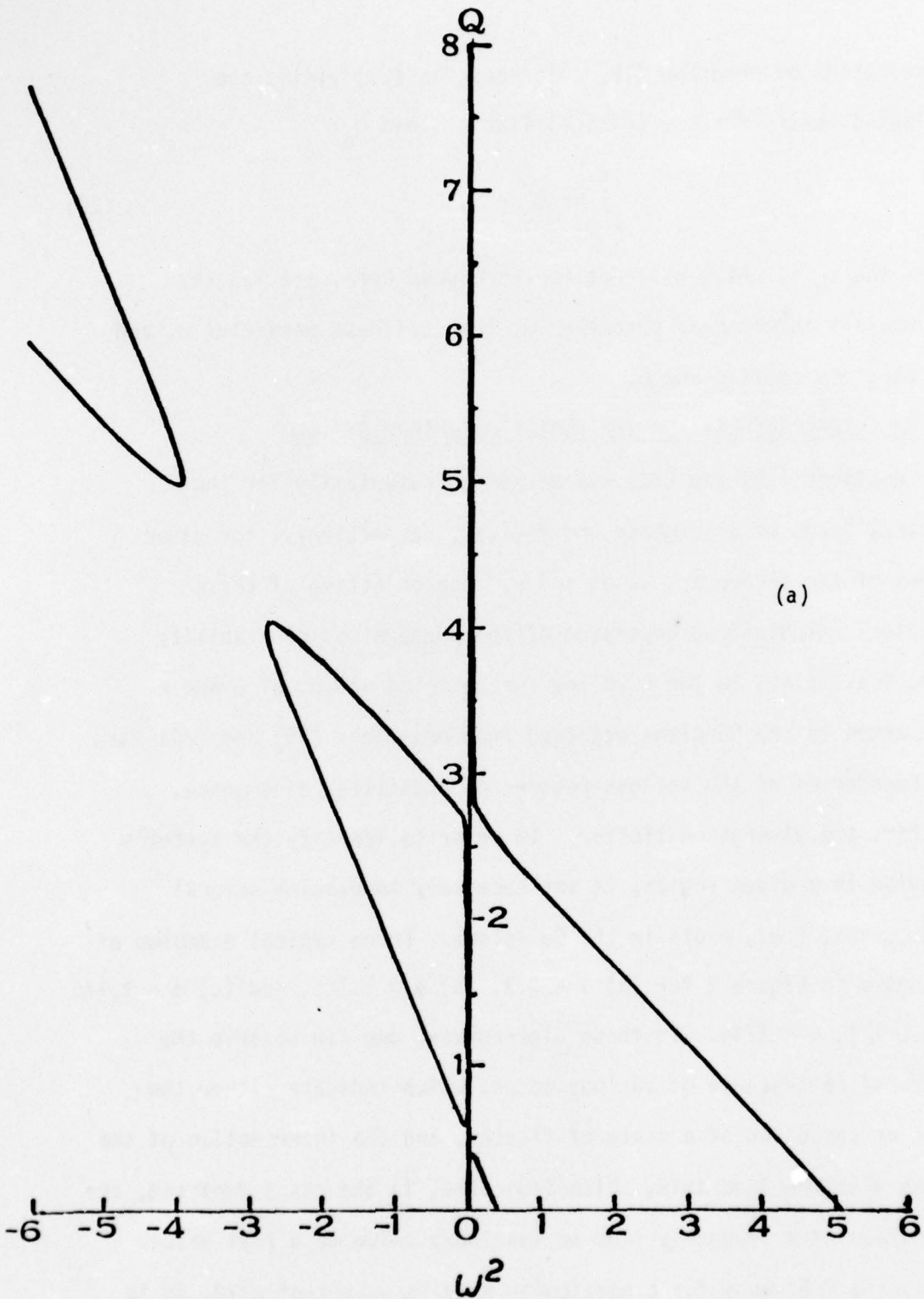
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Figure 2. Eigencurves for $\mu = 1$, $\kappa = 0.1$, and (a) $\alpha = 0.3$, (b) $\alpha = 1.125$, and (c) $\alpha = 1.175$ (1 of 3).
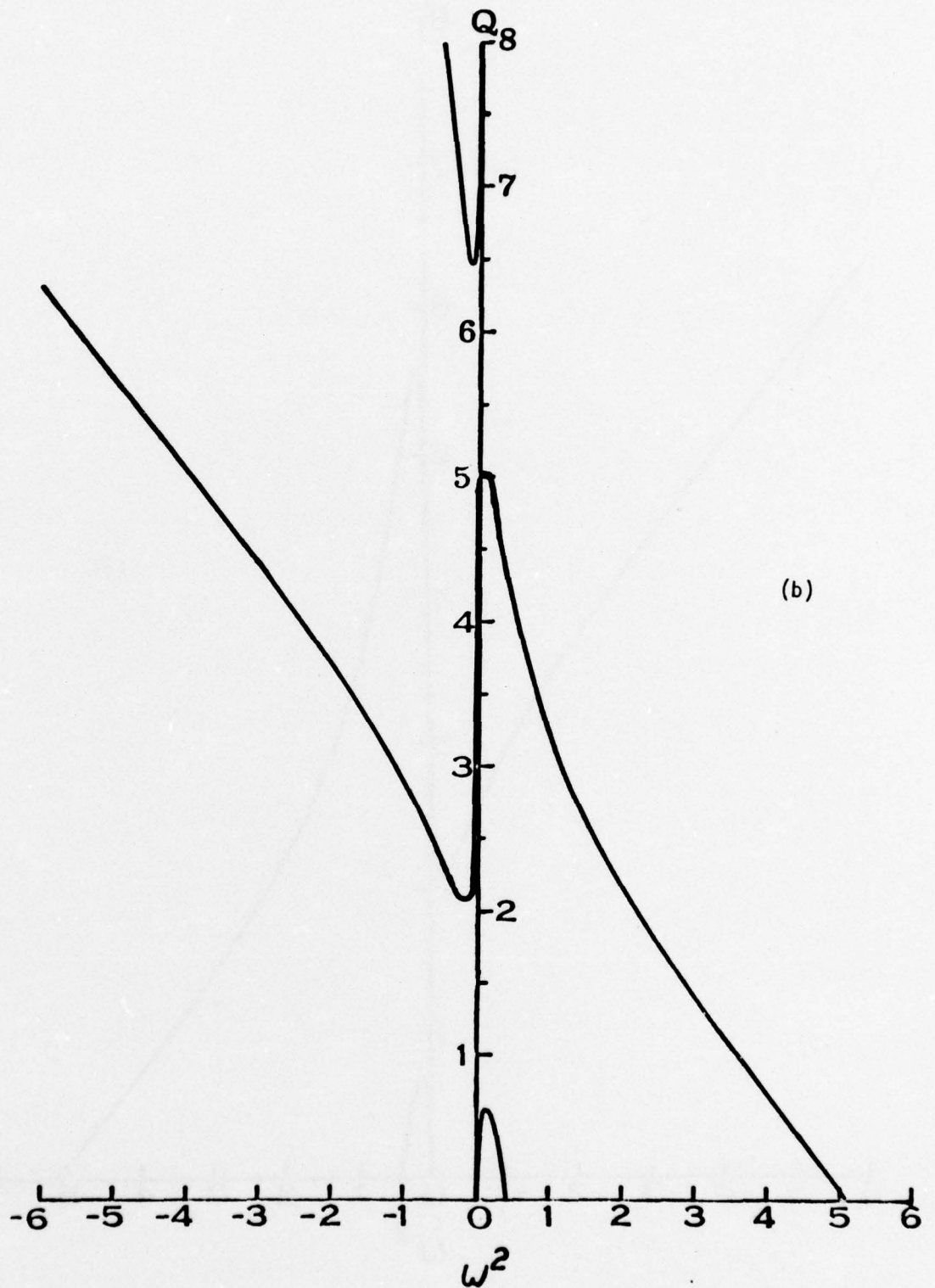
(b)

Figure 2. Eigencurves for $\mu = 1$, $\kappa = 0.1$, and (a) $\alpha = 0.3$, (b) $\alpha = 1.125$, and (c) $\alpha = 1.175$ (2 of 3).
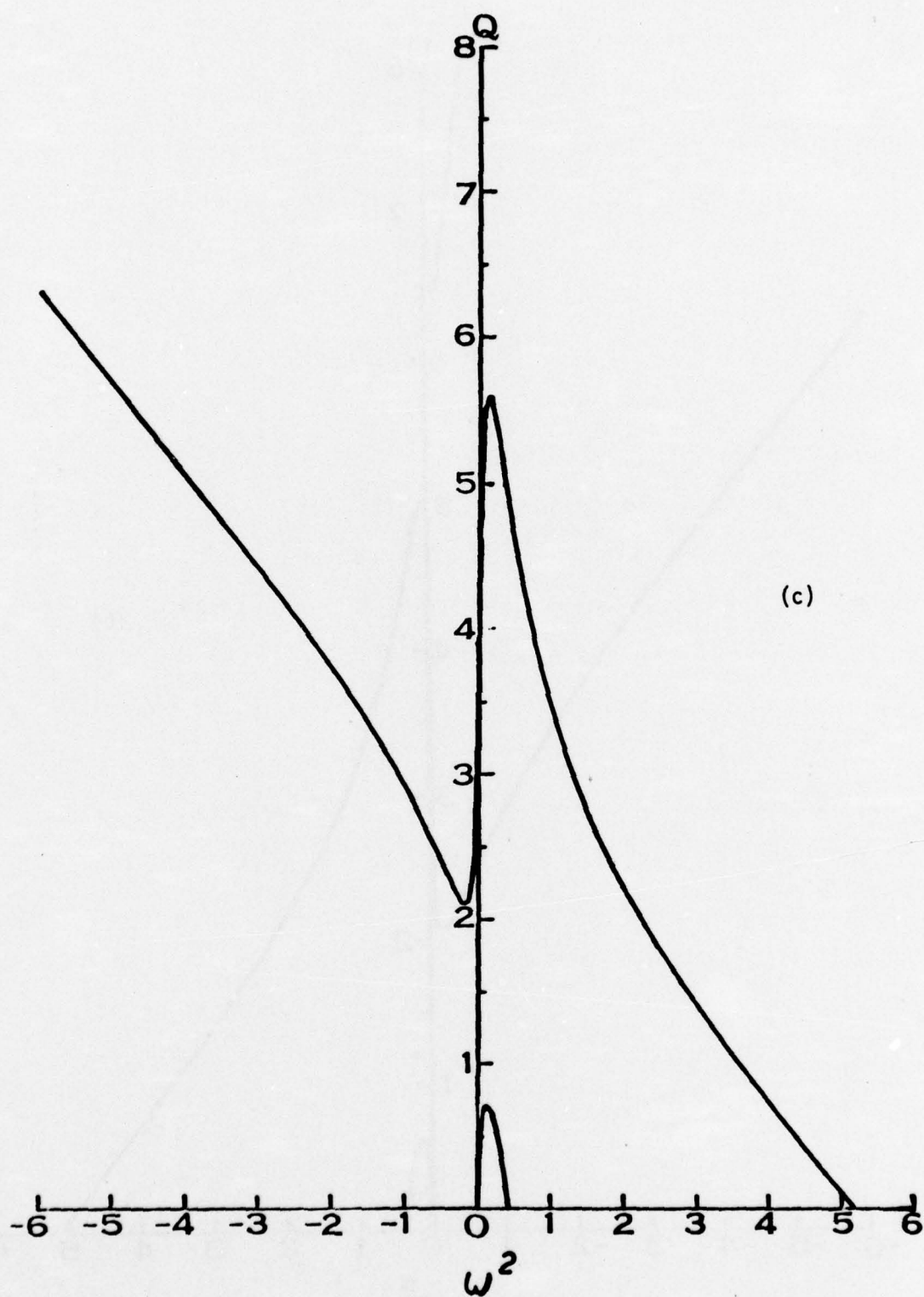
Figure 2. Eigencurves for $\mu = 1$, $\kappa = 0.1$, and (a) $\alpha = 0.3$, (b) $\alpha = 1.125$, and (c) $\alpha = 1.175$ (3 of 3).

understood that this mode grows exponentially in time (divergence).
If, however, it happens that ω^2 has a complex value for some mode, the
motion in this mode consists of an oscillation with an exponentially
growing amplitude (flutter). It may be observed from Figure 2c that
$\omega_1^2 < 0$ and ω_2^2 ω_3^2 are complex numbers when Q>5.615. In this situation,
the various modes of the system simultaneously exhibit instability by
both flutter and divergence.

Stability maps for κ = 1/10, 1, and 10 with μ = 1 are shown in
Figures 3, 4, and 5, respectively. The various regions of stability
and instability are labeled as follows: stable region (SR), flutter
region (FR), divergence region (DR), and flutter-divergence region
(FDR). These figures reveal that the shape and extent of the primary
flutter region is strongly dependent upon the value of the spring
stiffness coefficient κ. For relatively weakly supported bases, as in
the cases of Figures 3 and 4, the flutter region is fairly large and
extends beyond the range of the tangency coefficient, α, shown here.
However, as the spring stiffness is increased, say to κ = 10 as in
Figure 5, the flutter region contracts significantly in size and is
bounded by a closed curve. As the value of κ is raised still further,
the shape and location of the flutter region in the Qα-plane approach
those reported by Herrmann and Bungay [1] for a double pendulum
mounted on a rigid base.

In Figures 6 and 7, the stability maps have been plotted for
μ = 1/10 and μ = 10, respectively, with κ held fixed at κ = 1. Comparing
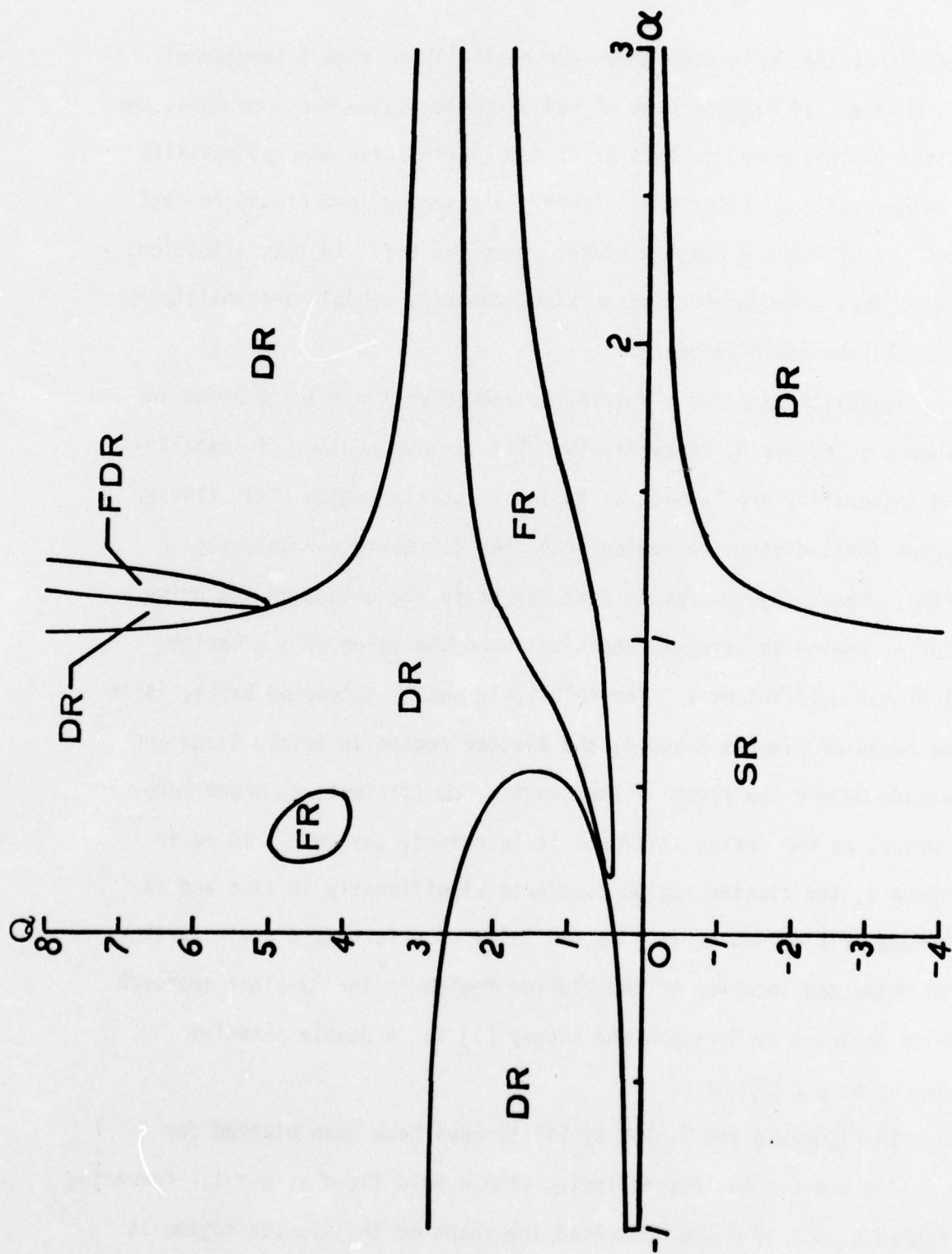Figures 6, 4, and 7 one sees that the shape of the flutter region is

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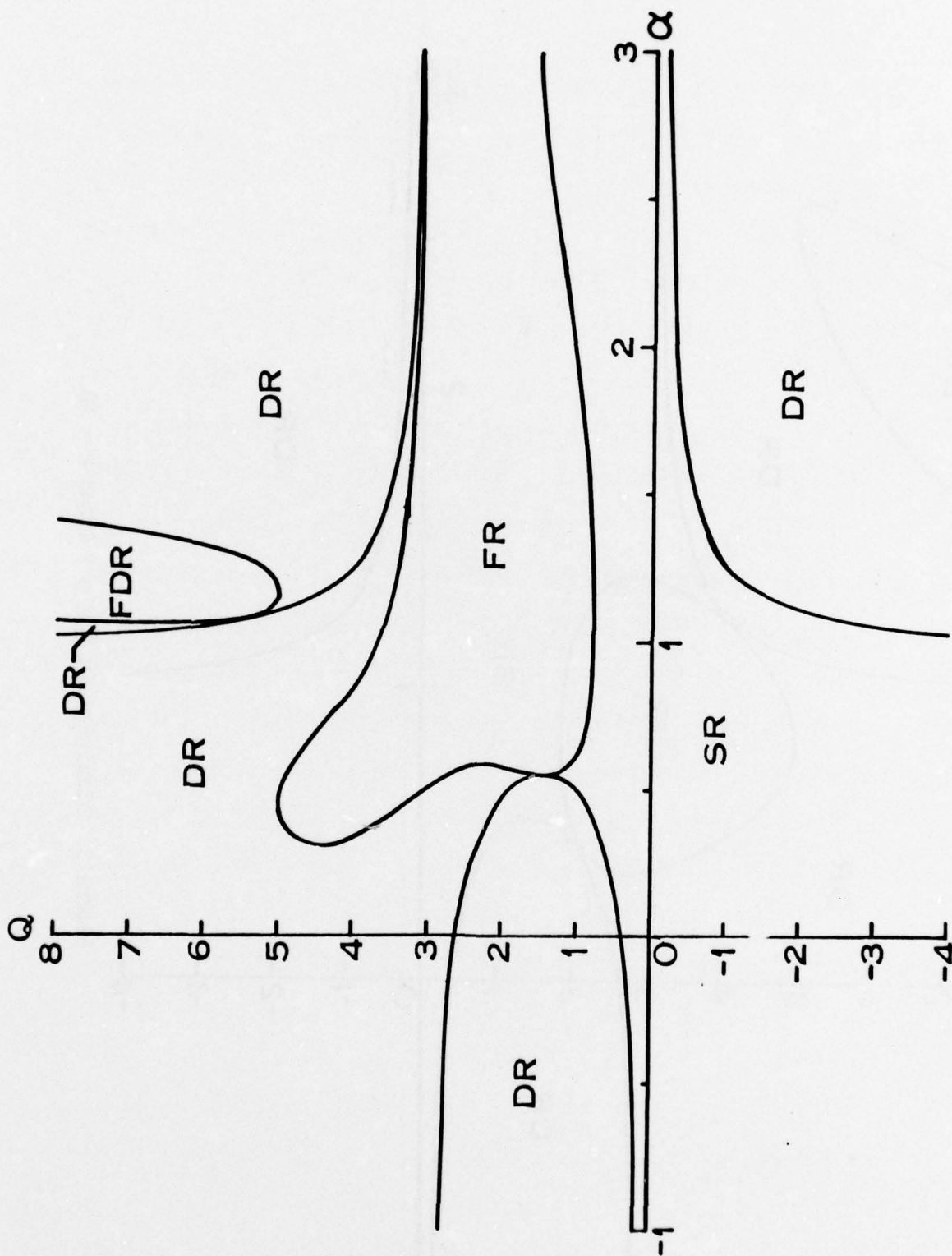Figure 3. Stability map for $\mu = 1$ and $\kappa = 0.1$.

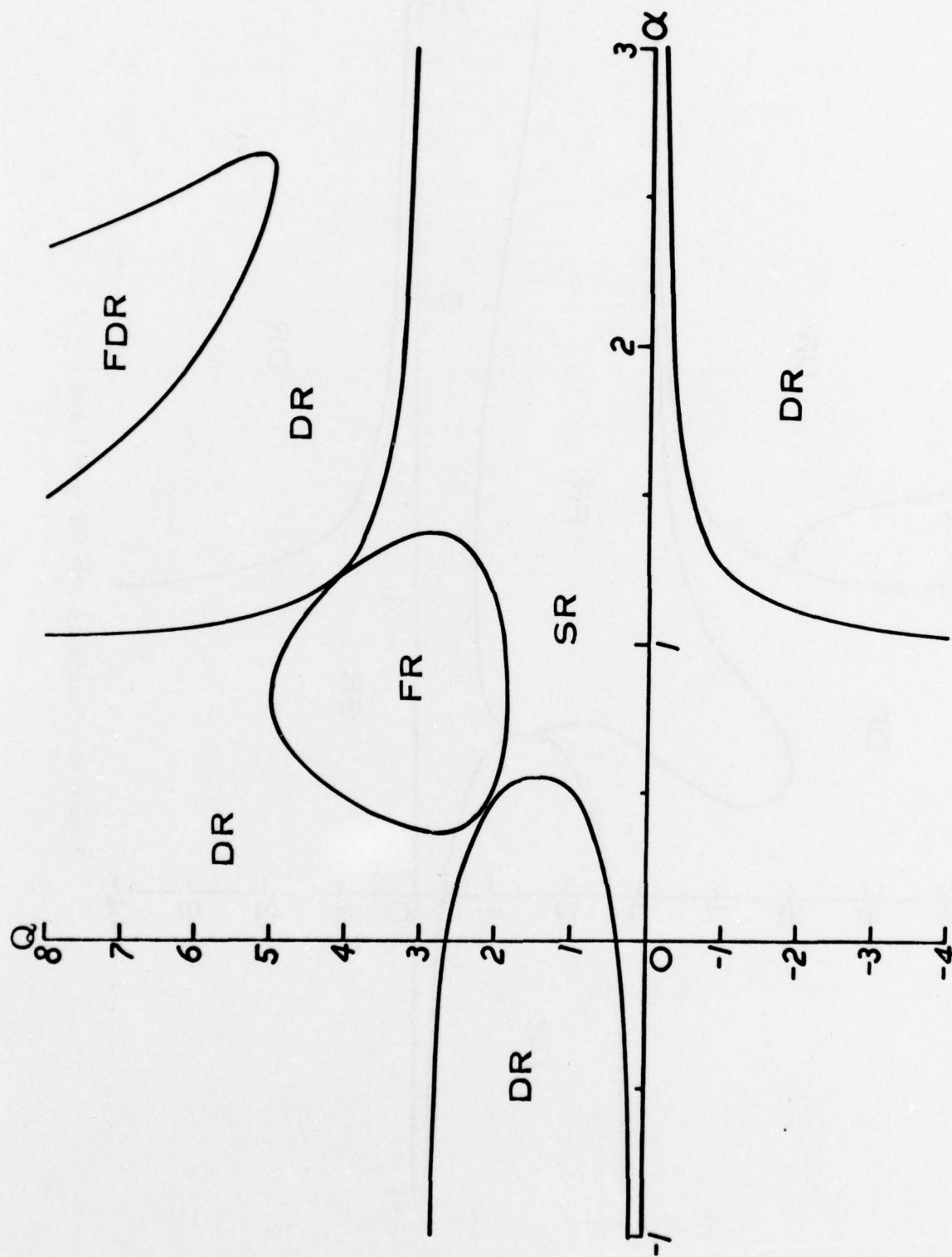Figure 4. Stability map for $\mu = 1$ and $\kappa = 1$.

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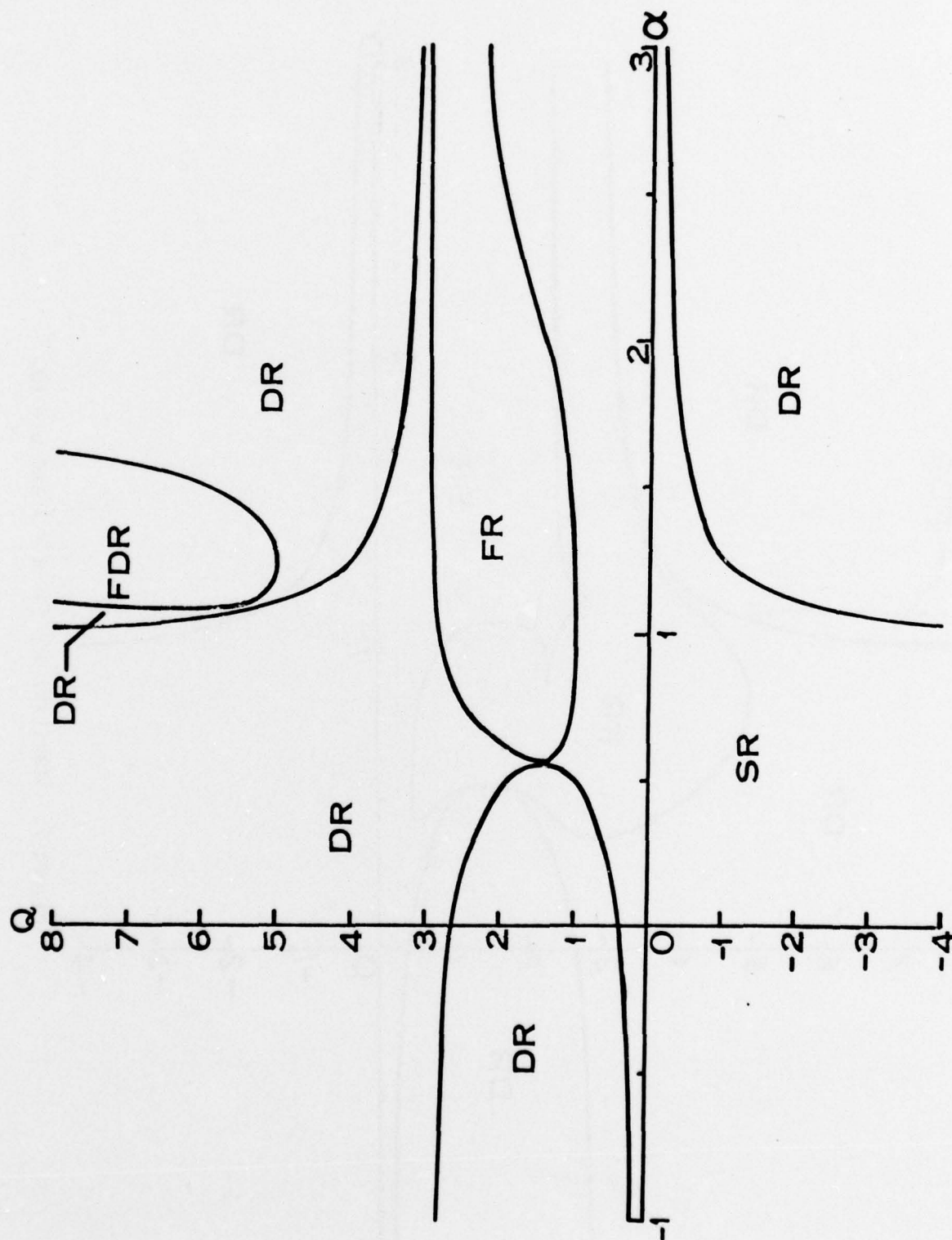Figure 5. Stability map for μ = 1 and κ = 10.

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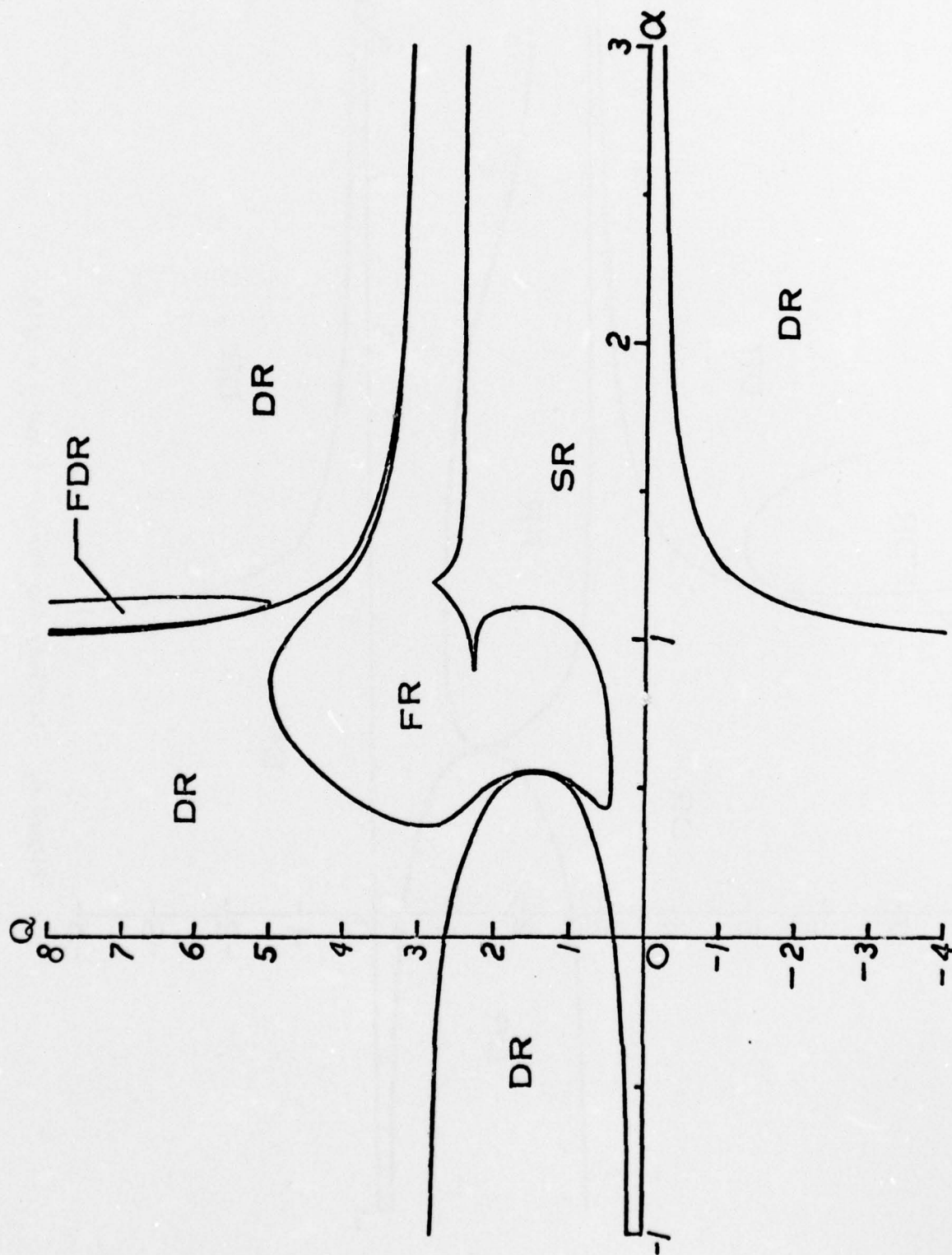Figure 6. Stability map for $\kappa = 1$ and $\mu = 1/10$.

Figure 7. Stability map for $\kappa = 1$ and $\mu = 10$.

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significantly influenced by the value of the mass parameter μ.

5. <u>NUMERICAL RESULTS FOR A QUASI-DYNAMIC SYSTEM</u>

It is of some interest to examine the system of equations (10) - (12) under the supposition that the mass of the base is negligibly small. Since the determinant of the dimensionless inertia matrix $\underset{\sim}{A}$ is det $(\underset{\sim}{A}) = \mu_1\mu_2\mu$, it is obvious that det $(\underset{\sim}{A}) = 0$ when $\mu = 0$. Therefore, in the case of a massless base, the system under consideration becomes an example of a quasi-dynamic system [3], whose motion is described by the differential equations

$$(\mu_1 + \mu_2)\ddot{x} + \kappa x + (\mu_1 + \mu_2)\ddot{\phi}_1 + \mu_2\ddot{\phi}_2 + \alpha Q\phi_2 = 0, \qquad (23)$$

$$(\mu_1 + \mu_2)\ddot{x} + (\mu_1 + \mu_2)\ddot{\phi}_1 + (2-Q)\phi_1 + \mu_2\ddot{\phi}_2 - (1-\alpha Q)\phi_2 = 0 \quad (24)$$

$$\mu_2\ddot{x} + \mu_2\ddot{\phi}_1 - \phi_1 + \mu_2\ddot{\phi}_2 + [1 - (1-\alpha)Q]\phi_2 = 0, \qquad (25)$$

which are obtained from equations (10) - (12) upon setting $\mu = 0$. Moreover, because $\underset{\sim}{A}$ is singular, it follows, according to reference [3], that an "internal constraint" must exist. This is easily identified simply by subtracting equation (24) from equation (23), the result being

$$\kappa x = (2-Q)\phi_1 - \phi_2 . \qquad (26)$$

Using equation (26) to eliminate the $\ddot{x}$ terms in equations (23) and (25), one finds a pair of equations of the form

$$\sum_{n=1}^{2} [A_{mn}\ddot{\phi}_n + (C_{mn} + QD_{mn})\phi_n] = 0, \quad m = 1,2, \qquad (27)$$

3. Ku, A. B., "On the Stability of a Linear Nongyroscopic Conservative System," <u>Zeitschrift für angewandte Mathematic und Physik</u>, Vol 20, 1977, pp. 986-991.

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where now

$$A_{11} = (\mu_1 + \mu_2)(2 + \kappa - Q), \qquad A_{12} = (\kappa - 1)\mu_2 - \mu_1,$$

$$A_{21} = \mu_2(2 + \kappa - Q), \qquad A_{22} = \mu_2(\kappa - 1),$$

$$C_{11} = 2\kappa, \quad C_{12} = -\kappa, \qquad D_{11} = -\kappa, \quad D_{12} = \kappa\alpha \qquad (28)$$

$$C_{21} = -\kappa, \quad C_{22} = \kappa, \qquad D_{21} = 0, \quad D_{22} = (\kappa - 1).$$

It should be noted here that the new dimensionless inertia matrix $\underset{\sim}{A}$, some of whose elements Amn, which are defined in equation (28), depend linearly upon Q, is not symmetric. In addition, the value of its determinant is

$$\det (\underset{\sim}{A}) = \kappa\mu_1\mu_2(2 + \kappa - Q) . \qquad (29)$$

Assuming a solution for equation (27) of the type given in equation (13), one can verify that the frequency equation for the quasi-dynamic system is simply

$$p_2\omega^4 - p_4\omega^2 + p_6 = 0, \qquad (30)$$

where p_2, p_4, and p_6 are obtained from equation (17) upon setting $\mu = 0$. Equivalently, equation (30) can be expressed as

$$(A + QH)\omega^4 + (D + BQ + IQ^2)\omega^2 + CQ^2 + EQ + F = 0, \qquad (31)$$

where

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$$A = \mu_1\mu_2 (2 + \kappa), \quad H = -\mu_1\mu_2, \quad D = [(1 + \kappa)\mu_1 + (1 + 5\kappa)\mu_2],$$

$$B = (2 + \kappa)[(1 - \alpha)\mu_1 + \mu_2] + \mu_1 + (1 + \kappa)\mu_2, \quad I = (a - 1)\mu_1 - \mu_2,$$

$$C = \kappa(1 - \alpha), \quad E = -3\kappa(1 - \alpha), \quad F = \kappa . \tag{32}$$

It may be remarked at this point that, if the H and I terms were deleted from equation (31), the resulting frequency-load relationship would be such that the eigencurves would be conic sections in the $Q\omega^2$-plane, as reported in reference [1]. However, in the present investigation, the values of H and I are generally different from zero and, therefore, the eigencurves are not conic sections. It may be anticipated, then, that the present system may possess some flutter characteristics that are unusual relative to those described in reference [1].

The critical divergence loads can be obtained once again from the condition $p_6 = 0$, i.e., from equation (19), that results when $\omega = 0$ is substituted into equation (30). On the other hand, for the flutter loads, the condition for the coalescence of the frequencies obtained from equation (30) is $p_4^2 - 4p_2p_6 = 0$, whence

$$I^2Q^4 + 2(IB - 2HC)Q^3 + (B^2 + 2ID - 4HE - 4AC)Q^2 +$$

$$+ 2(BD - 2HF - 2AE)Q + D^2 - 4AF = 0, \tag{33}$$

which is a quartic polynomial in Q.

In the case of $\mu_1 = 2$ and $\mu_1 = 1$, equation (32) becomes

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1. Herrmann, G., and Bungay, R. W., "On the Stability of Elastic Systems Subjected to Nonconservative Forces," Journal of Applied Mechanics, Vol 31, 1964, pp. 435-440.

$$A = 2(2 + \kappa), \quad H = -2, \quad D = -(3 + 7\kappa),$$

$$B = 9 + 4\kappa - 2\alpha (2 + \kappa), \quad I = 2\alpha - 3,$$

$$C = \kappa(1 - \alpha), \quad E = -3\kappa(1 - \alpha), \quad F = \kappa .$$

Consequently, equations (31) and (33) can be expressed as

$$2(2 + \kappa - Q)\omega^4 - \{3 + 7\kappa + [2(2 + \kappa) - 9 - 4\kappa]Q + (3 - 2\alpha)Q^2\}\omega^2 +$$
$$+ \kappa(1 - \alpha)Q^2 - 3\kappa(1 - \alpha)Q + \kappa = 0 \qquad (34)$$

and

$$(2\alpha - 3)^2 Q^4 - 2[4(2 + \kappa)(1 - \alpha)^2 + 2(7 + \kappa)(1 - \alpha) + 5 + 2\kappa]Q^3 +$$
$$+ [4(2 + \kappa)^2(1 - \alpha)^2 + 4(13 + 6\kappa)(1 - \alpha) + 31 + 34\kappa + 4\kappa^2]Q^2 -$$
$$- 2[2(\kappa + 2)(\kappa + 3)(1 - \alpha) + (2\kappa + 1)(7\kappa + 15)]Q + 9 + 26\kappa + 41\kappa^2 = 0,$$

respectively. $\qquad\qquad (35)$

Solving the frequency equation (34) for ω^2 as a function of Q for given values of α and κ, one can construct eigencurves. Two such curves, plotted for $\alpha = 1.05$ and $\alpha = 1.25$ with $\kappa = 1$, are shown in Figure 8. In Figure 8a, the frequencies of the two modes of vibration coalesce in the first quadrant of the $Q\omega^2$-plane at Q = 1.01. Consequently, the system becomes unstable initially by flutter. As the value of Q is raised even higher, flutter eventually gives way to divergence which persists over the remainder of the range of Q shown. In the case of $\alpha = 1.25$ (Figure 8b), the onset of flutter occurs at Q = 1.02. Flutter persists for 1.02<Q<2.78, and in the interval 2.78<Q<4.01 the system is divergent. However, for 4.01<Q<5.02 the system is stable once more, although flutter now occurs a second time, in this instance at Q = 5.02, as the result of a second coalescence of frequencies.
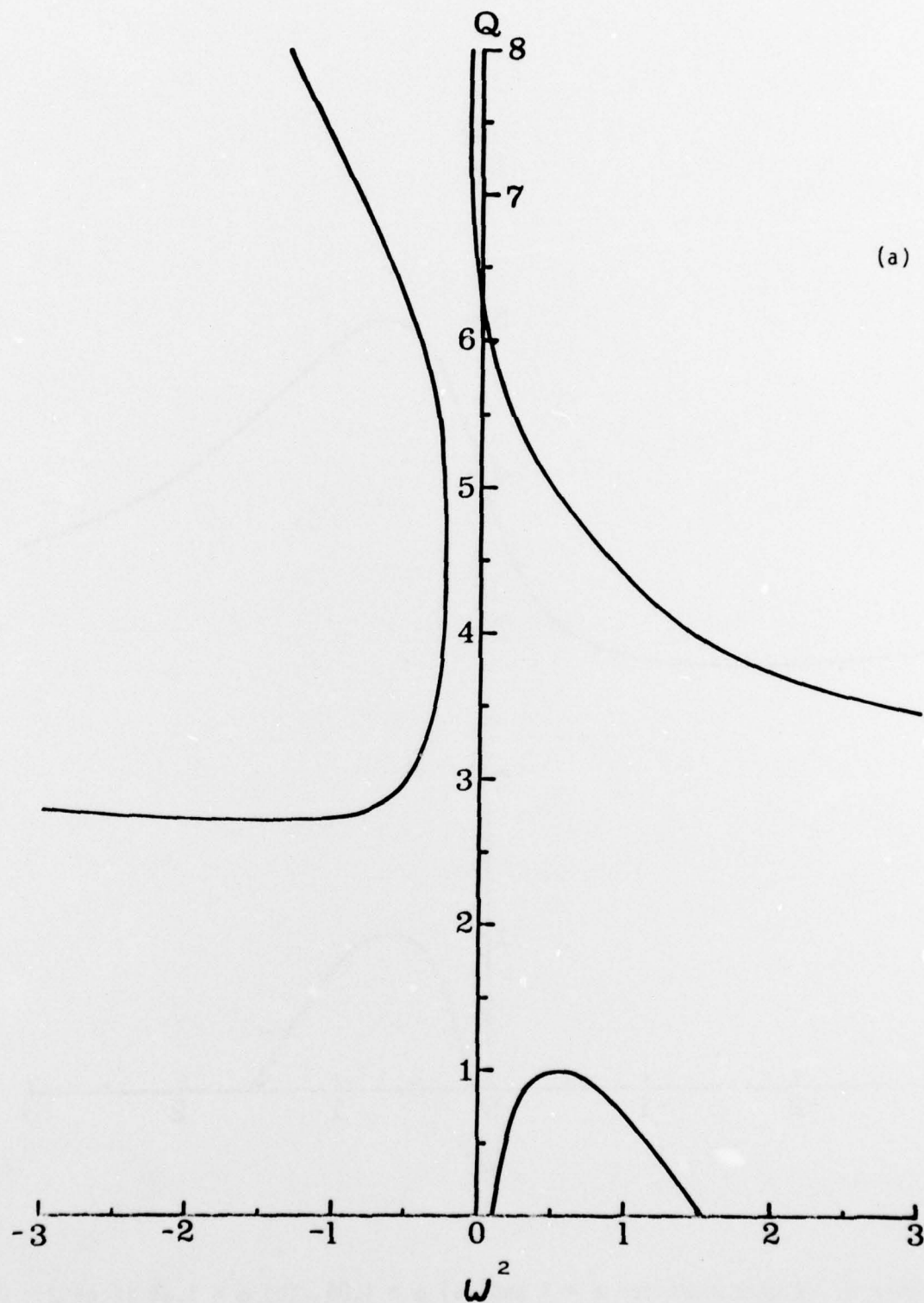
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Figure 8. Eigencurves for $\kappa = 1$ and (a) $\alpha = 1.05$, (b) $\alpha = 1.25$ (1 of 2).
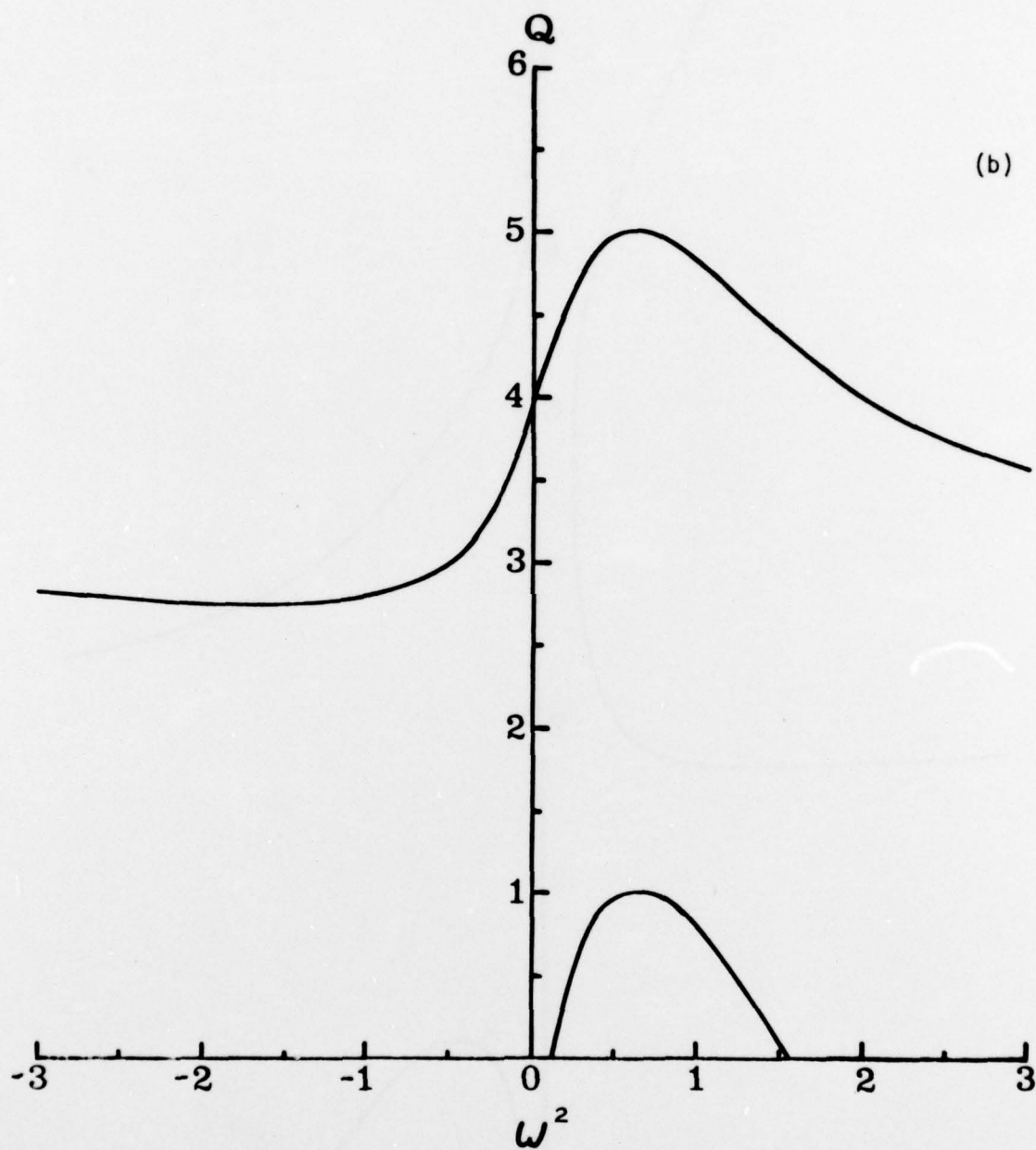
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Figure 8. Eigencurves for $\kappa = 1$ and (a) $\alpha = 1.05$, (b) $\alpha = 1.25$ (2 of 2).

Performing computations with equation (35) for the critical flutter loads and using the data obtained earlier from equation (19) for the critical divergence loads, one can plot stability maps in the $Q\alpha$-plane. In Figures 9, 10, and 11, some results are shown for $\kappa = 1/10$, 1, and 2. Although these figures resemble the stability maps in Figures 3, 4, and 5 for the purely dynamic system, it may be observed that the flutter-divergence region in a stability map for the purely dynamical system becomes a flutter region in the corresponding stability map for the quasi-dynamic system. In addition, the divergence region in the upper right corner in each of the Figures 9-11 is replaced by a stable region in the quasi-dynamic case.
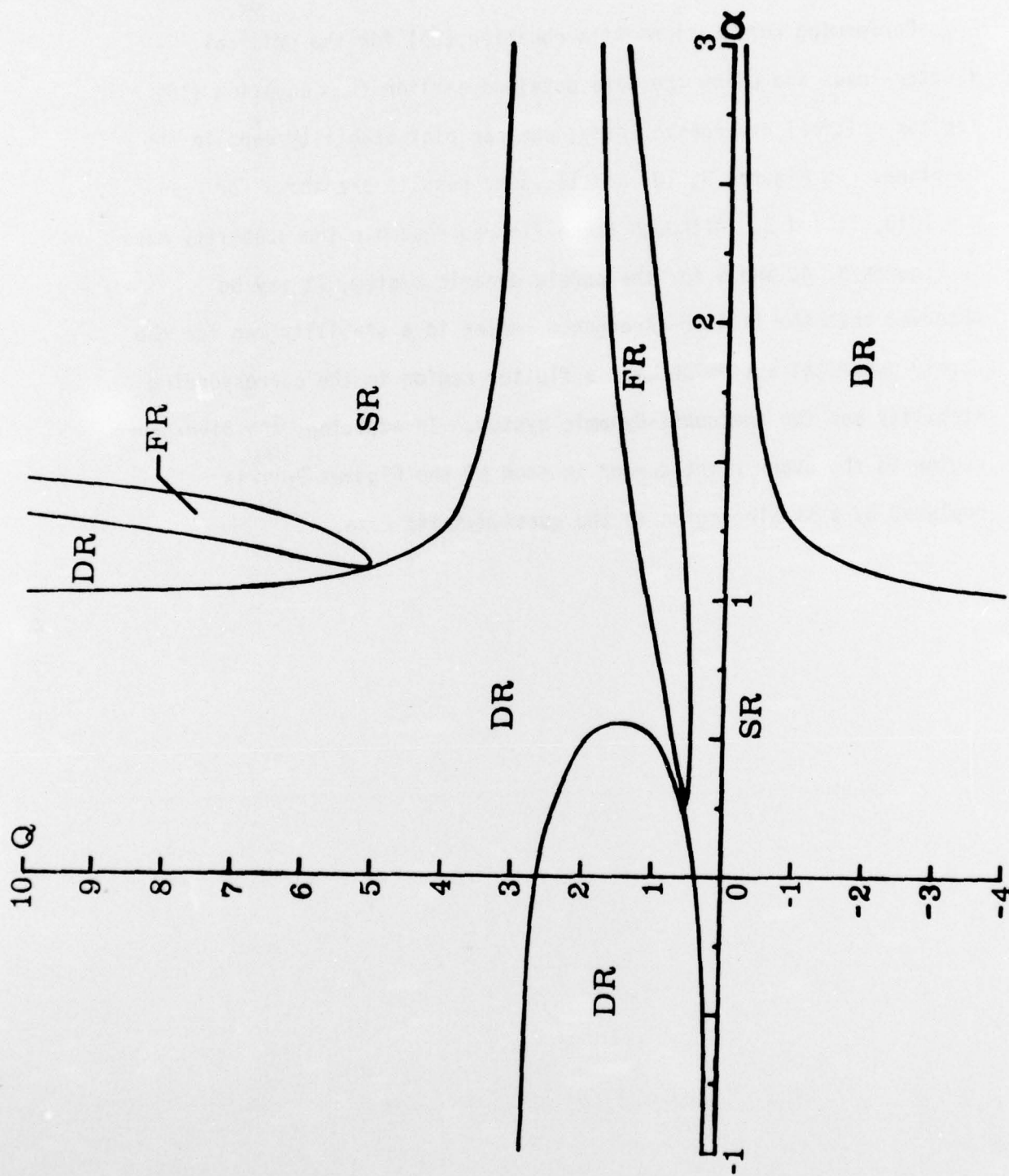
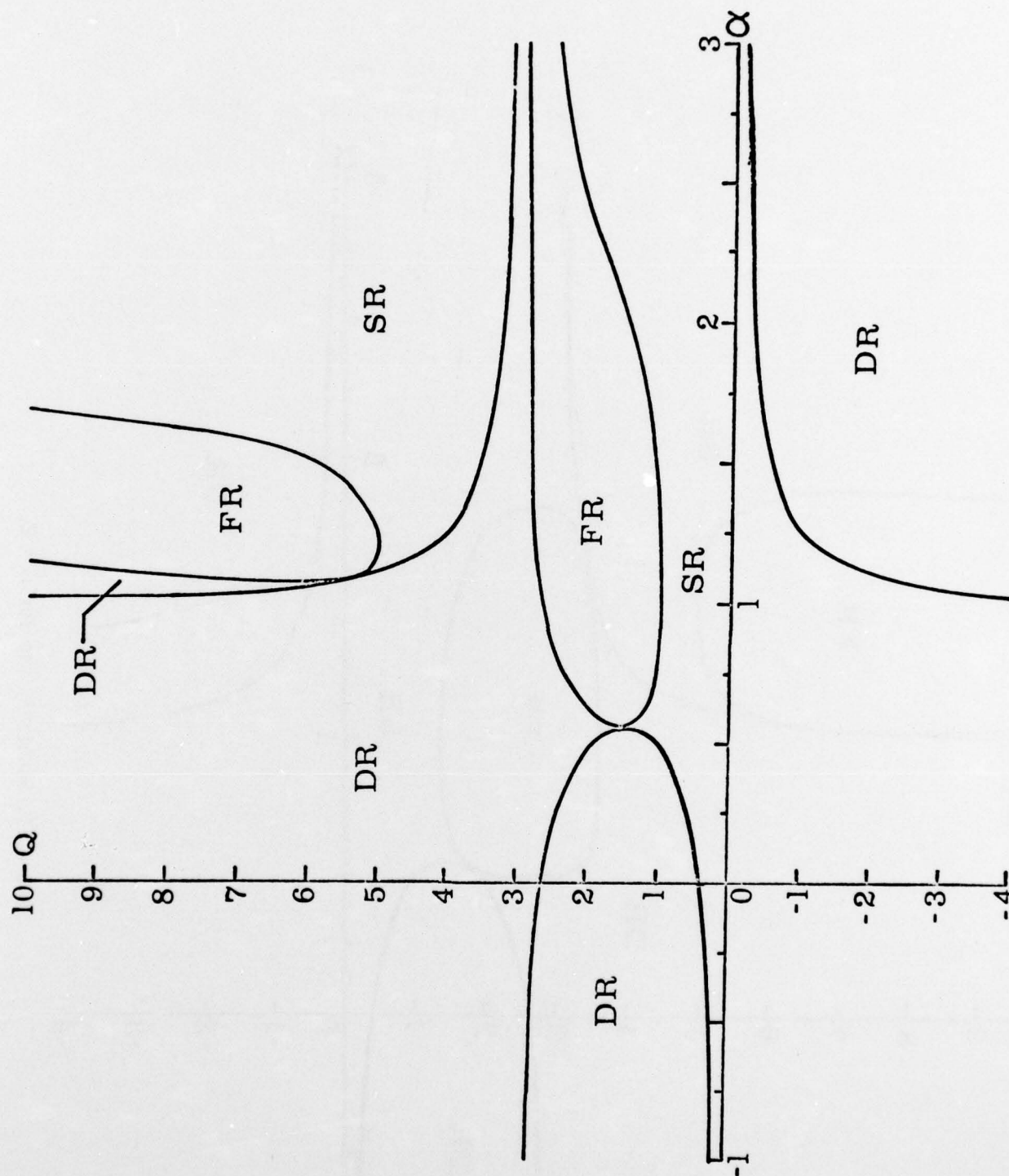Figure 9. Stability map for $\kappa = 1/10$.
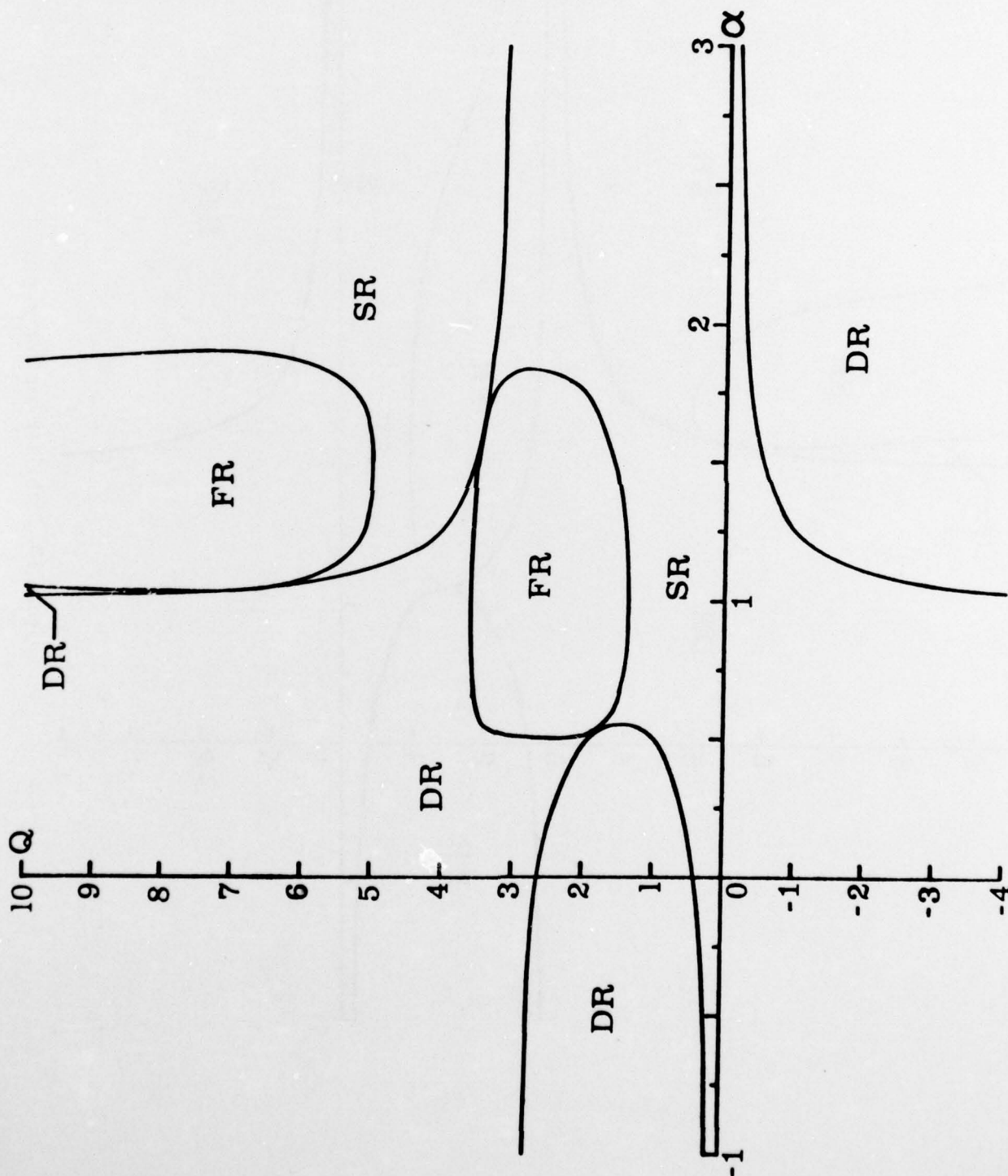
Figure 10. Stability map for $\kappa = 1$.

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Figure 11. Stability map for κ = 2.

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